DAY SIXTEEN

Oscillations

Learning & Revision for the Day

- Periodic Motion
- Simple Harmonic Motion
- Force and Energy in SHM
- Simple Pendulum
- Oscillations of a Spring
- Free, Damped, Forced and Resonant Vibrations

Periodic Motion

A motion which repeats itself over a regular interval of time is called a periodic motion. A periodic motion in which a body moves back and forth repeatedly about a fixed point (called mean position) is called **oscillatory** or **vibratory motion.**

Displacement as a Function of Time

In a periodic motion, each displacement value is repeated after a regular interval of time, displacement can be represented as a function of time $y = f(t)$.

Periodic Function

A function which repeats its value after a fix interval of time is called a periodic function.

$$y(t) = y(t + T)$$

where, T is the period of the function.

Trigonometric functions $\sin\theta$ and $\cos\theta$ are simplest periodic functions having period of 2π.

Simple Harmonic Motion

Simple Harmonic Motion (SHM) is that type of periodic motion in which the particle moves to and fro or back and forth about a fixed point under a restoring force, whose magnitude is directly proportional to its displacement

i.e. $$F \propto x \text{ or } F = -kx$$

where, k is a positive constant called the **force constant** or **spring factor** and x is displacement.

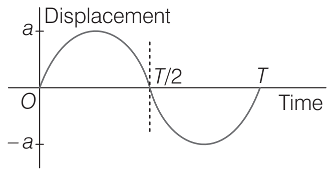
Differential equations of SHM, for linear SHM, $\frac{d^2y}{dt^2} + \omega^2 y = 0$,

for angular SHM, $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

Terms Related to SHM

The few important terms related to simple harmonic motion are given as

- **Displacement** The displacement of a particle executing SHM is, in general, expressed as $y = A\sin(\omega t + \phi)$.



  where, A is the amplitude of SHM, ω is the angular frequency $\left(\text{where } \omega = \frac{2\pi}{T} = 2\pi\nu\right)$ and ϕ is the initial phase of SHM. However, displacement may also be expressed as $x = A\cos(\omega t + \phi)$.

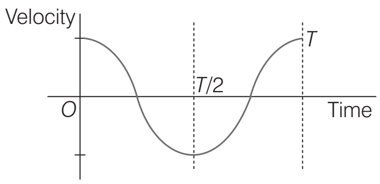
- **Amplitude** The maximum displacement on either side of mean position is called amplitude of SHM.

- **Velocity** The velocity of a particle executing SHM at an instant is defined as the time rate of change of its displacement at that instant.

  Velocity, $\frac{dy}{dt} = v = \omega\sqrt{A^2 - y^2}$

  At the mean position ($y = 0$), during its motion $v = A\omega = v_{max}$ and at the extreme positions ($y = \pm A$), $v = 0$.

  Velocity amplitude $= v_{max} = A\omega$



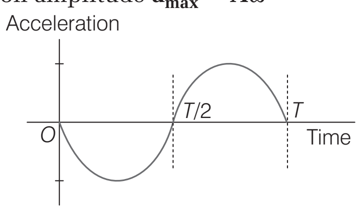
- **Acceleration** The acceleration of a particle executing SHM at an instant is defined as the time rate of change of velocity at that instant.

  Acceleration, $\frac{d^2y}{dt^2} = a = -\omega^2 y$

  The acceleration is also a variable.

  At the mean position ($y = 0$), acceleration $a = 0$ and at the extreme position ($y = \pm A$), the acceleration is $a_{max} = -A\omega^2$.

  $\therefore$ Acceleration amplitude $a_{max} = A\omega^2$



- **Phase** Phase is that physical quantity which tells about the position and direction of motion of any particle at any moment. It is denoted by ϕ.

- **Phase Difference** If two particles perform S.H.M and their equations are

  $y_1 = a\sin(\omega t + \phi_1)$ and $y_2 = a\sin(\omega t + \phi_2)$

  phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

- **Time Period** The time taken by a particle to complete one oscillation is called time period. It is denoted by T.

  $\therefore$ Time period of SHM,

  $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{|y|}{|a|}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

- **Frequency and Angular Frequency** It is defined as the number of oscillations executed by body per second. SI unit of frequency is hertz.

  Angular frequency of a body executing periodic motion is equal to product of frequency of the body with factor 2π. Angular frequency, $\omega = 2\pi n$.

Force and Energy in SHM

- **Force** For an object executing SHM, a force always acts on it, which tries to bring it in mean position, i.e. it is always directed towards mean position.

  The equation of motion, $\mathbf{F} = m\mathbf{a}$,

  $$\therefore \quad F = -m\omega^2 x \qquad [\because a = -\omega^2 x]$$

  $$= -kx \qquad \left[\because \omega = \sqrt{\frac{k}{m}}\right]$$

  Here, negative sign shows that direction of force is always opposite to the direction of displacement.

- **Energy** If a particle of mass m is executing SHM, then at a displacement x from the mean position, the particle possesses potential and kinetic energy.

  At any displacement x,

  Potential energy, $U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}k\,x^2$

  Kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$

  Total energy, $E = U + K = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 m\nu^2 A^2$

  If there is no friction, the total mechanical energy, $E = K + U$, of the system always remains constant even though K and U change.

Simple Pendulum

A simple pendulum, in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string. The motion of a simple pendulum is simple harmonic for very small angular displacement (α) whose time period and frequency are given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{and } \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

where, l is the effective length of the string and g is acceleration due to gravity.

- If a pendulum of length l at temperature $\theta°$C has a time period T, then on increasing the temperature by $\Delta\theta°$C its time period changes to $T \times \Delta T$,

  where, $\frac{\Delta T}{T} = \frac{1}{2}\alpha\,\Delta\theta$

where, α is the temperature coefficient of expansion of the string.

- A second's pendulum is a pendulum whose time period is 2s. At a place where $g = 9.8\ ms^{-2}$, the length of a second's pendulum is 0.9929 m (or 1 m approx).
- If the bob of a pendulum (having density ρ) is made to oscillate in a non-viscous fluid of density σ, then it can be shown that the new period is

$$T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{\sigma}{\rho}\right)}}$$

- If a pendulum is in a lift or in some other carriage moving vertically with an acceleration a, then the effective value of the acceleration due to gravity becomes $(g \pm a)$ and hence

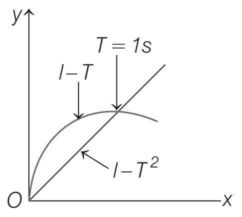
$$T = 2\pi\sqrt{\frac{l}{(g \pm a)}}$$

Here, positive sign is taken for an upward accelerated motion and negative sign for a downward accelerated motion.

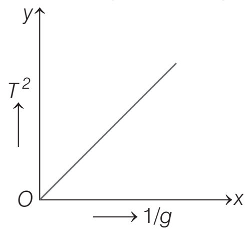
- If a pendulum is made to oscillate in a freely falling lift or an orbiting satellite, then the effective value of g is zero and hence, the time period of the pendulum will be infinity and therefore pendulum will not oscillate at all.
- If the pendulum bob of mass m has a charge q and is oscillating in an electrical field E acting vertically downwards, then $T = 2\pi\sqrt{\frac{l}{\left(g \pm \frac{qE}{m}\right)}}$
- If pendulum of charge q is oscillating in an electric field E acting horizontally, then

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{q^2E^2}{m^2}}}}$$

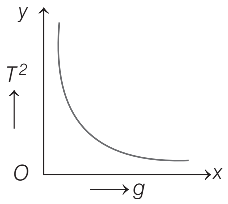
- The graphs l–T and l–T^2 intersect at $T = 1$ s.



- The graph between T^2 and $1/g$ is a straight line.

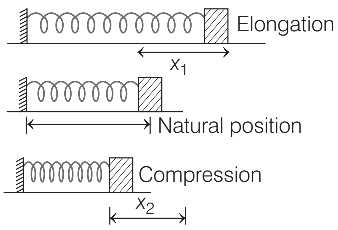


- The graph between T^2 and g is a rectangular hyperbola.



Oscillations of a Spring

If the mass is once pulled, so as to stretch the spring and is then released, then a restoring force acts on it which continuously tries to restore its mean position.



Restoring force $F = -kl$,

where k is force constant and l is the change in length of the spring.

Here, $x_1 = x_2 = l$

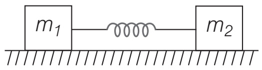
- The spring pendulum oscillates simple harmonically having time period and frequency given by

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ and } \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- If the spring is not light but has a mass m_s, then

$$T = 2\pi\sqrt{\frac{m + 1/3\, m_s}{k}}$$

- If two masses m_1 and m_2, connected by a spring, are made to oscillate on a horizontal surface, then its period will be



$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

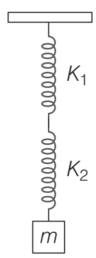
where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass of the system.

Combination of Springs

- If two springs of spring constants k_1 and k_2 are joined in **series** (horizontally and vertically), then their equivalent spring constant k_s is given by $\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$

$$\Rightarrow \quad k_s = \frac{k_1 k_2}{k_1 + k_2}$$

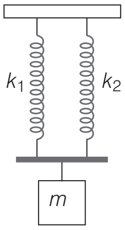
$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$



(a)

- If the two springs of spring constants k_1 and k_2 are joined in **parallel** as shown, then their equivalent spring constant $k_p = k_1 + k_2$

  hence, $T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$



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Free, Damped, Forced and Resonant Vibrations

Some of the vibrations are described below

Free Vibrations

If a body, capable of oscillating is slightly displaced from its position of equilibrium and then released, it starts oscillating with a frequency of its own.

Such oscillations are called free vibrations.The frequency with which a body oscillates is called the **natural frequency** and is given by

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, a body continues to oscillate with a constant amplitude and a fixed frequency.

Damped Vibrations

The oscillations in which the amplitude decreases gradually with the passage of time are called damped vibrations.

Damping force, $F_d = -bv$

where, v is the velocity of the oscillator and b is a damping constant.

The displacement of the oscillator is given by

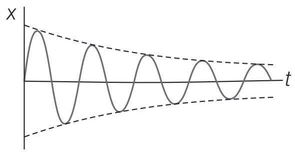
$$x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

where, ω' = the angular frequency

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

The mechanical energy E of the oscillator is given by

$$E(t) = \frac{1}{2}kA^2e^{-bt/m}$$



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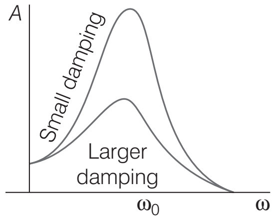

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Forced Vibrations

The vibrations in which a body oscillates under the effect of an external periodic force, whose frequency is different from the natural frequency of the oscillating body, are called forced vibrations.

In forced vibrations, the oscillating body vibrates with the frequency of the external force and amplitude of oscillations is generally small.

Resonant Vibrations

It is a special case of forced vibrations in which the frequency of external force is exactly same as the natural frequency of the oscillator.

As a result, the oscillating body begins to vibrate with a large amplitude leading to the phenomenon of resonance to occur. Resonant vibrations play a very important role in music and in tuning of station/channel in a radio/TV, etc.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 An elastic ball is dropped from a certain height and returns to the same height after elastic collision on the floor. What is the nature of repeated motion of the ball?

(a) Simple harmonic, oscillatory and periodic
(b) Simple harmonic, oscillatory but not periodic
(c) Simple harmonic, periodic, but not oscillatory
(d) Oscillatory, periodic but not simple harmonic

2 The displacement of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$

(a) The motion is oscillatory but not SHM
(b) The motion is SHM with amplitude $a + b$
(c) The motion is SHM with amplitude $a^2 + b^2$
(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

3 The graph between restoring force and time in case of SHM is

(a) a straight line (b) a circle
(c) a parabola (d) a sine curve

4 Out of the following functions representing motion of a particle which represents SHM? ➜ CBSE AIPMT 2011

I. $y = \sin \omega t - \cos \omega t$ II. $y = \sin^3 \omega t$

III. $y = 5 \cos \left(\frac{3\pi}{4} - 3\omega t\right)$ IV. $y = 1 + \omega t + \omega^2 t^2$

(a) Only (IV) does not represent SHM
(b) (I) and (III)
(c) (I) and (II)
(d) Only (I)

5 The displacement of SHM is given by

$$y = 5 \cos (10t + 0.6)$$

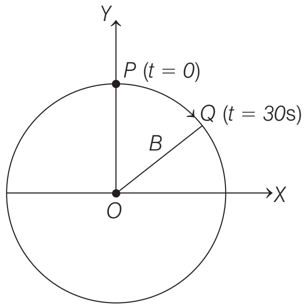
What is the initial phase of the SHM?

(a) 5 rad (b) 10 rad
(c) 0.6 rad (d) None of these

6 A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a) $\frac{T}{4}$ (b) $\frac{T}{8}$
(c) $\frac{T}{12}$ (d) $\frac{T}{2}$

7 Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is



(a) $x(t) = B \sin\left(\frac{2\pi t}{30}\right)$ (b) $x(t) = B \cos\left(\frac{\pi t}{15}\right)$
(c) $x(t) = B \sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$ (d) $x(t) = B \cos\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

8 The displacement of a particle is represented by the equation $y = 3 \cos \left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is

(a) simple harmonic with period $2\pi/\omega$
(b) simple harmonic with period π/ω
(c) periodic but not simple harmonic
(d) non-periodic

9 When two displacements represented by $y_1 = a \sin(\omega t)$ and $y_2 = b \cos(\omega t)$ are superimposed the motion is ➜ CBSE AIPMT 2015

(a) not a simple harmonic
(b) simple harmonic with amplitude $\frac{a}{b}$
(c) simple harmonic with amplitude $\sqrt{a^2 + b^2}$
(d) simple harmonic with amplitude $\frac{(a + b)}{2}$

10 The displacement of a particle along the X-axis is given by $x = a \sin^2 \omega t$. The motion of the particle corresponds to ➜ CBSE AIPMT 2010

(a) simple harmonic motion of frequency $\frac{\omega}{\pi}$
(b) simple harmonic motion of frequency $\frac{3\omega}{2\pi}$
(c) non-simple harmonic motion
(d) simple harmonic motion of frequency $\frac{\omega}{2\pi}$

11 The time period of a SHM is 16 s. It starts its motion from the equilibrium position. After 2 s its velocity is $\pi \text{ ms}^{-1}$. What is its displacement amplitude?

(a) $\sqrt{2}$ m (b) $2\sqrt{2}$ m
(c) $4\sqrt{2}$ m (d) $8\sqrt{2}$ m

12 The maximum velocity of a simple harmonic motion represented by $y = 3\sin\left(100t + \frac{\pi}{6}\right)$ is given by

(a) 300 (b) $\frac{3\pi}{6}$ (c) 100 (d) $\frac{\pi}{6}$

13 A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are v_1 and v_2, respectively. Its time period is ➜ CBSE AIPMT 2015

(a) $2\pi\sqrt{\frac{x_1^2 + x_2^2}{v_1^2 + v_2^2}}$ (b) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

(c) $2\pi\sqrt{\frac{v_1^2 + v_2^2}{x_1^2 + x_2^2}}$ (d) $2\pi\sqrt{\frac{v_1^2 - v_2^2}{x_1^2 - x_2^2}}$

14 A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a and time period T. The speed of the pendulum at $x = \frac{a}{2}$ will be ➜ CBSE AIPMT 2009

(a) $\frac{\pi a\sqrt{3}}{2T}$ (b) $\frac{\pi a}{T}$ (c) $\frac{3\pi^2 a}{T}$ (d) $\frac{\pi a\sqrt{3}}{T}$

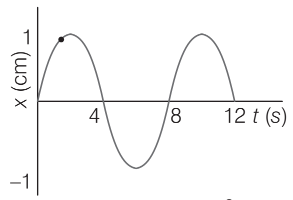
15 The amplitude of a particle executing SHM is 4 cm. At the mean position, the speed of the particle is 16 cm/s. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$ cm/s will be

(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm (c) 1 cm (d) 2 cm

16 If at any time the displacement of a simple pendulum be 0.02 m, then its acceleration is 2 ms^{-2}. What is the angular speed of the pendulum at that instant?

(a) 100 $rads^{-1}$ (b) 10 $rads^{-1}$ (c) 1 $rads^{-1}$ (d) 0.1 $rads^{-1}$

17 The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is



(a) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (b) $\frac{-\pi^2}{32}$ cm/s^2

(c) $\frac{\pi^2}{32}$ cm/s^2 (d) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

18 A particle moves such that its acceleration, a is given by $a = -bx$, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is

(a) $2\pi\sqrt{b}$ (b) $2\pi/\sqrt{b}$ (c) $2\pi/b$ (d) $2\sqrt{\pi/b}$

19 The oscillation of a body on a smooth horizontal surface is represented by the equation, $x = A\cos(\omega t)$

where, x = displacement at time t

ω = frequency of oscillation

Which one of the following graphs shows correctly the variation a with t? ➜ CBSE AIPMT 2014

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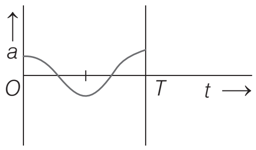

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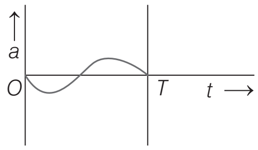

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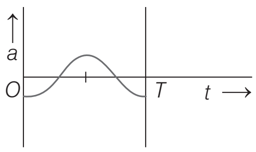

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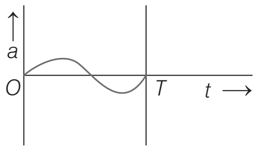


Here, a = acceleration at time t

T = Time period

20 Which one of the following equations of motion represents simple harmonic motion? ➜ CBSE AIPMT 2009

(a) Acceleration $= -k_0 x + k_1 x^2$
(b) Acceleration $= -k(x + a)$
(c) Acceleration $= k(x + a)$
(d) Acceleration $= kx$
(where, k, k_0, k_1 and a are all positive.)

21 Two simple harmonic motions of angular frequency 100 rad s^{-1} and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is

(a) 1 : 10 (b) 1 : 10^2 (c) 1 : 10^3 (d) 1 : 10^4

22 A system is subjected to two SHMs given by

$y_1 = 6\cos\omega t$ and $y_2 = 8\cos\omega t$

The resultant amplitude of SHM is given by

(a) 2 (b) 10 (c) 14 (d) 20

23 A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m/s^2 at a distance of 5 m from the mean position. The time period of oscillation is ➜ NEET 2018

(a) 2 s (b) π s (c) 2π s (d) 1 s

24 A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then, its time period in seconds is ➜ NEET 2017

(a) $\frac{\sqrt{5}}{\pi}$ (b) $\frac{\sqrt{5}}{2\pi}$

(c) $\frac{4\pi}{\sqrt{5}}$ (d) $\frac{2\pi}{\sqrt{3}}$

25 A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cms^{-1}. The frequency of its oscillation is

(a) 3 Hz (b) 2 Hz (c) 4 Hz (d) 1 Hz

26 A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then, its time period of vibration will be ➜ CBSE AIPMT 2015

(a) $\frac{\beta^2}{\alpha^2}$ (b) $\frac{\alpha}{\beta}$

(c) $\frac{\beta^2}{\alpha}$ (d) $\frac{2\pi\beta}{\alpha}$

27 A particle executes linear simple hormonic motion with an amplitude of 2 cm. When the particle is at 1 cm from the mean position the magnitude of its velocity is equal to that of its acceleration. Then, its time period in second is

(a) $\frac{1}{2\pi\sqrt{3}}$ (b) $2\pi\sqrt{3}$ (c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2\pi}$

28 A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a \sin\left(\omega t + \frac{\pi}{6}\right)$.

After the elapse of what fraction of the time period, the velocity of the point will be equal to half of its maximum velocity ?

(a) $\frac{T}{8}$ (b) $\frac{T}{6}$ (c) $\frac{T}{3}$ (d) $\frac{T}{12}$

29 A particle executes SHM with a period of T second and amplitude A metre. The shortest time it takes to reach point $\frac{A}{\sqrt{2}}$ metre from its mean position in seconds is

(a) T (b) $\frac{T}{4}$ (c) $\frac{T}{8}$ (d) $\frac{T}{16}$

30 A SHM has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = \frac{A}{2}$ is

(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$

31 The total energy of SHM is E. What will be the kinetic energy of the particle, when displacement is half of the amplitude?

(a) $\frac{\sqrt{3}}{2}E$ (b) $\frac{E}{2}$ (c) $\frac{3E}{4}$ (d) $\frac{E}{3}$

32 A body executing SHM has amplitude of 4 cm. What is the distance at which the body has equal value of both KE and PE?

(a) $2\sqrt{2}$ cm (b) $1/\sqrt{2}$ cm (c) $\sqrt{2}$ cm (d) $\sqrt{2}/6$ cm

33 If the length of a pendulum is quadrupled, its time period is

(a) quadrupled (b) halved
(c) doubled (d) unchanged

34 A tunnel is bored along the diameter of the earth and a stone is dropped into it. What happens to the stone?

(a) It oscillates between the two ends of the tunnel
(b) It comes to rest at the centre of the earth
(c) It will go out of the other end of the tunnel
(d) It will come to a permanent stop at the other end of the tunnel

35 Two simple pendulums first of bob mass M_1 and length L_1, second of bob mass M_2 and length L_2. $M_1 = M_2$ and $L_1 = 2L_2$. If the vibrational energies of both are same. Then, which is correct? ➜ AIIMS 2010

(a) Amplitude of B greater than A
(b) Amplitude of B smaller than A
(c) Amplitude will be same
(d) None of the above

36 The potential energy of a simple harmonic oscillator when the particle is half way to its end point is ➜ CBSE AIPMT 2003

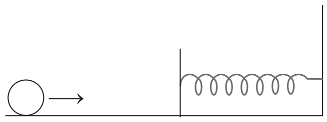
(a) $\frac{1}{4}E$ (b) $\frac{1}{2}E$ (c) $\frac{2}{3}E$ (d) $\frac{1}{8}E$

(where, E is the total energy)

37 A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is ➜ NEET 2016

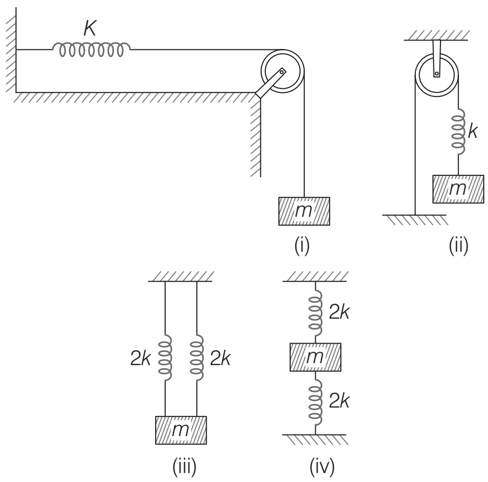
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{16}{9}$ (d) $\frac{9}{16}$

38 A mass of 0.5 kg moving with a speed of 1.5 ms^{-1} on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50\ Nm^{-1}$. The maximum compression of the spring would be



(a) 0.15 m (b) 0.12 m (c) 1.5 m (d) 0.5 m

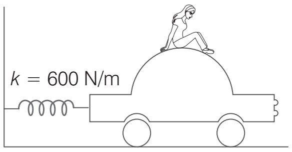
39 A block of mass m is suspended by different springs of force constant shown in figure.



Let time period of oscillation in these four positions be T_1, T_2, T_3 and T_4. Then, which of the following statement is correct?

(a) $T_1 = T_2 = T_4$ (b) $T_1 = T_2$ and $T_3 = T_4$
(c) $T_1 = T_2 = T_3$ (d) $T_1 = T_3$ and $T_2 = T_4$

40 A girl is sitting on the roof of a flat toy car of mass 6 kg. If no slipping takes place between car and the girl, then what should be the mass of the child in order to have period of system equal to 0.758 s?

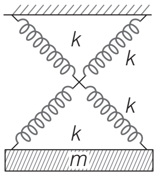


(a) 9 kg (b) 2.74 kg (c) 6 kg (d) 7.28 kg

41 Two springs of force constants k_1 and k_2 are connected in series. The spring constant of the combination is

(a) $k_1 + k_2$ (b) $\frac{k_1 + k_2}{2}$ (c) $\frac{k_1 + k_2}{k_1 k_2}$ (d) $\frac{k_1 k_2}{k_1 + k_2}$

42 As shown in figure, a simple harmonic motion oscillator having identical four springs has time period.



(a) $T = 2\pi\sqrt{\frac{m}{4k}}$ (b) $T = 2\pi\sqrt{\frac{m}{2k}}$

(c) $T = 2\pi\sqrt{\frac{m}{k}}$ (d) $T = 2\pi\sqrt{\frac{2m}{k}}$

43 When a string is divided into three segments of lengths l_1, l_2 and l_3, the fundamental frequencies of these three segments are ν_1, ν_2 and ν_3, respectively. The original fundamental frequency (ν) of the string is

➜ CBSE AIPMT 2012

(a) $\sqrt{\nu} = \sqrt{\nu_1} + \sqrt{\nu_2} + \sqrt{\nu_3}$ (b) $\nu = \nu_1 + \nu_2 + \nu_3$

(c) $\frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}$ (d) $\frac{1}{\sqrt{\nu}} = \frac{1}{\sqrt{\nu_1}} + \frac{1}{\sqrt{\nu_2}} + \frac{1}{\sqrt{\nu_3}}$

44 One-fourth length of a spring of force constant k is cut away. The force constant of the remaining spring will be

(a) $\frac{3}{4}k$ (b) $\frac{4}{3}k$ (c) k (d) $4k$

45 The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be

➜ CBSE AIPMT 2010

(a) T (b) $\frac{T}{\sqrt{2}}$ (c) $2T$ (d) $\sqrt{2}T$

46 A spring is vibrating with frequency n under some mass. If it is cut into two equal parts and same mass is suspended, then the new frequency is

(a) $n/2$ (b) n (c) $n\sqrt{2}$ (d) $n/\sqrt{2}$

47 The amplitude of a damped oscillator becomes half in 1 min. The amplitude after 3 min will be $\frac{1}{X}$ times the original, where X is

(a) 2×3 (b) 2^3 (c) 3^2 (d) 3×2^2

48 The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are

➜ CBSE AIPMT 2012

(a) kg ms^{-1} (b) kg ms^{-2}
(c) kg s^{-1} (d) kg s

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 Nm^{-1}. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan? (take, $g = 10\ ms^{-2}$)

(a) 8.0 cm
(b) 10.0 cm
(c) Any value less than 12.0 cm
(d) 4.0 cm

2 The equations of two linear SHM's are

$x = a \sin \omega t$, along X-axis

$y = a \sin 2\omega t$, along Y-axis

If they act on a particle simultaneously, the trajectory of the particle is

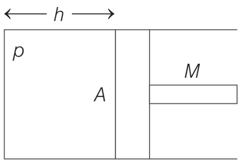
(a) $\frac{y^2}{a^2} + \frac{x^2}{4a^2} = 1$ (b) $y^2 = \frac{4x^2}{a^2}(a^2 - x^2)$

(c) $y^2 = 2ax$ (d) None of these

3 A tunnel is made along a chord inside the earth and a ball is released in it. What will be the time period of oscillation of the ball?

(a) $2\pi\sqrt{\frac{R}{2g}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{\frac{2R}{g}}$ (d) $\pi\sqrt{\frac{R}{g}}$

4 A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. Its period is



(a) $T = 2\pi\sqrt{\dfrac{Mh}{pA}}$ (b) $T = 2\pi\sqrt{\dfrac{MA}{ph}}$

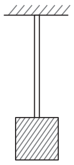
(c) $T = 2\pi\sqrt{\dfrac{M}{pAh}}$ (d) None of these

5 From the ceiling of a train, a pendulum of length l is suspended. The train is moving with an acceleration a_0 on horizontal surface. The period of oscillation of the pendulum is

(a) $T = 2\pi\sqrt{\dfrac{l}{g}}$ (b) $T = 2\pi\sqrt{\dfrac{l}{\sqrt{a_0^2 + g^2}}}$

(c) $T = \pi\sqrt{\dfrac{l}{\sqrt{a_0^2 + g^2}}}$ (d) $T = \pi\sqrt{\dfrac{l}{g}}$

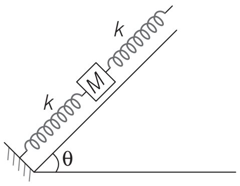
6 A wire of length l, area of cross-section A and Young's modulus of elasticity Y is suspended from the roof of a building. A block of mass m is attached at lower end of the wire. If the block is displaced from its mean position and released, then the block starts oscillating. Time period of these oscillations will be



(a) $2\pi\sqrt{\dfrac{Al}{mY}}$ (b) $2\pi\sqrt{\dfrac{AY}{ml}}$

(c) $2\pi\sqrt{\dfrac{ml}{YA}}$ (d) $2\pi\sqrt{\dfrac{m}{YAl}}$

7 On a smooth inclined plane a body of mass M is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has a force constant k, the period of oscillation of the body is (assuming the spring as massless)



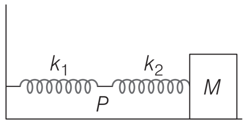
(a) $2\pi\sqrt{\dfrac{M}{2k}}$ (b) $2\pi\sqrt{\dfrac{2M}{k}}$

(c) $2\pi\sqrt{\dfrac{M\sin\theta}{2k}}$ (d) $2\pi\sqrt{\dfrac{2M\sin\theta}{k}}$

8 A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T. Then,

(a) $T \propto \sqrt{\rho}$ (b) $T \propto \dfrac{1}{\sqrt{A}}$ (c) $T \propto \dfrac{1}{\rho}$ (d) $T \propto \dfrac{1}{\sqrt{m}}$

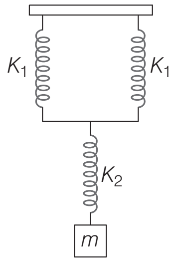
9 The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\dfrac{k_1 A}{k_2}$ (b) $\dfrac{k_2 A}{k_1}$

(c) $\dfrac{k_1 A}{k_1 + k_2}$ (d) $\dfrac{k_2 A}{k_1 + k_2}$

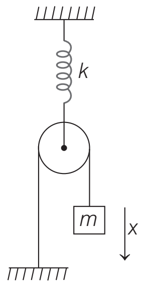
10 What will be the force constant of the spring system as shown in the figure?



(a) $\dfrac{k_1}{2} + k_2$ (b) $\left[\dfrac{1}{2k_1} + \dfrac{1}{k_2}\right]^{-1}$

(c) $\dfrac{1}{2k_1} + \dfrac{1}{k_2}$ (d) $\left[\dfrac{2}{k_1} + \dfrac{1}{k_1}\right]^{-1}$

11 In the given figure, the spring has a force constant k. The pulley is light and smooth the spring and string are light. The suspended block has a mass m. If the block is slightly displaced from its equilibrium position and then released. The period of its vertical oscillation is



(a) $2\pi\sqrt{\dfrac{m}{k}}$ (b) $4\pi\sqrt{\dfrac{m}{k}}$

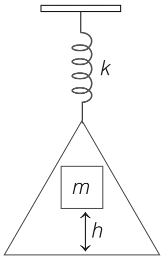
(c) $2\pi\sqrt{\dfrac{k}{m}}$ (d) $4\pi\sqrt{\dfrac{k}{m}}$

12 The particle executing SHM has a kinetic energy $K_0\cos^2\omega t$. The maximum values of the potential energy and the total energy respectively are

(a) 0 and $2K_0$ (b) $\dfrac{K_0}{2}$ and K_0

(c) K_0 and $2K_0$ (d) K_0 and K_0

13 A load of mass m falls from a height h on to the scale pan hung from a spring as shown in figure. If the spring constant is k and mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is



(a) $\frac{mg}{k}$

(b) $\frac{mg}{k}\sqrt{1+\frac{2hk}{mg}}$

(c) $\frac{mg}{k}+\frac{mg}{k}\sqrt{1+\frac{2hk}{mg}}$

(d) None of these

14 The angular frequency and amplitude of a simple pendulum are ω and A, respectively. At the displacement y from the mean position, the kinetic energy is K and potential energy is U. What is the ratio of K/U?

(a) $MA^2\omega^2\sin^2\omega t$ (b) $MA^2\omega^2\cos^2\omega t$

(c) $(A^2-y^2)/y^2$ (d) $y^2/(A^2-y^2)$

15 The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm, the potential energy stored in it is

(a) $4U$ (b) $8U$ (c) $16U$ (d) $\frac{U}{4}$

16 A simple harmonic oscillator consists of a particle of mass m and an ideal spring with spring constant k. The particle oscillates with a time period T. The spring is cut into two equal parts. If one part oscillates with the same particle, the time period will be ➞ AIIMS 2012

(a) $2T$ (b) $\sqrt{2}T$ (c) $\frac{T}{\sqrt{2}}$ (d) $\frac{T}{2}$

17 A particle moves with simple hormonic motion in a straight line. In first τ s, after starting from rest it travels a distance a and in next τ s it travel $2a$, in same direction, then

(a) Amplitude of motion is τa
(b) Time period of oscillation is 8τ
(c) Amplitude of motion is $4a$
(d) Time period of oscillation is 6τ

ANSWERS

SESSION 1	1 (d)	2 (d)	3 (d)	4 (b)	5 (c)	6 (c)	7 (a)	8 (b)	9 (c)	10 (c)
	11 (d)	12 (a)	13 (b)	14 (d)	15 (d)	16 (b)	17 (d)	18 (b)	19 (c)	20 (b)
	21 (b)	22 (c)	23 (b)	24 (c)	25 (d)	26 (d)	27 (c)	28 (d)	29 (c)	30 (a)
	31 (c)	32 (a)	33 (c)	34 (a)	35 (b)	36 (a)	37 (d)	38 (a)	39 (b)	40 (b)
	41 (d)	42 (c)	43 (c)	44 (b)	45 (d)	46 (c)	47 (b)	48 (c)		
SESSION 2	1 (b)	2 (b)	3 (b)	4 (a)	5 (b)	6 (c)	7 (a)	8 (b)	9 (d)	10 (b)
	11 (b)	12 (d)	13 (b)	14 (c)	15 (c)	16 (c)	17 (d)			

Hints and Explanations

SESSION 1

1 Here, the ball moves under constant acceleration and hence displacement is not proportional to acceleration. So, the motion cannot be simple harmonic.

2 $y = a\sin\omega t + b\cos\omega t$ ...(i)

Let $a = A\cos\theta$ and $b = A\sin\theta$

Then,

$$a^2+b^2 = A^2(\cos^2\theta+\sin^2\theta) = A^2$$

or $A=\sqrt{a^2+b^2}$

From Eq. (i),

$y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t$
$= A\sin(\omega t+\theta)$

It is an equation of SHM with amplitude $A=\sqrt{a^2+b^2}$.

3 $F = Ma = M\omega^2 x = M\omega^2 A\sin\omega t$. Hence, graph between F and t is a sine curve.

4 For a simple harmonic motion,

$$a \propto \frac{d^2y}{dt^2} \propto -y$$

Hence, equations $y=\sin\omega t-\cos\omega t$ and $y = 5\cos\left(\frac{3\pi}{4}-3\omega t\right)$ are satisfying this condition and equation $y = 1+\omega t+\omega^2t^2$ is not periodic and $y=\sin^3\omega t$ is periodic but not simple hormonic motion.

5 In the equation of SHM, $y = A\cos(\omega t+\phi)$, the initial phase is the phase at $t=0$. Since, the phase is $\omega t+\phi$, therefore at $t=0$, we have $\phi = 0.6$ rad.

6 Let displacement equation of particle executing SHM is $y = a\sin\omega t$

As particle travels half of the amplitude from the equilibrium position, so

$$y=\frac{a}{2}$$

Therefore, $\frac{a}{2} = a\sin\omega t$

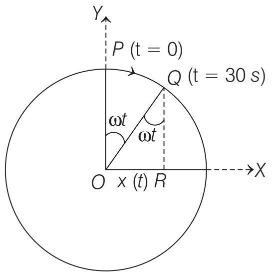
or $\sin\omega t = \frac{1}{2} = \sin\frac{\pi}{6}$ or $\omega t = \frac{\pi}{6}$

or $t = \frac{\pi}{6\omega}$ or $t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)}$ $\left[\text{as } \omega = \frac{2\pi}{T}\right]$

or $t=\frac{T}{12}$

Hence, the particle travels half of the amplitude from the equilibrium in $\frac{T}{12}$ s.

7 Given, $T = 30$ s, $OQ = B$. The projection of the radius vector on the diameter of the circle when a particle is moving with uniform angular velocity (ω) on a circle of reference is SHM. Let the particle go from P to Q in time t.



Then, $\angle POQ = \omega t = \angle OQR$. The projection of radius OQ on X-axis will be $OR = x(t)$ say.

In ΔOQR, $\sin\omega t = \dfrac{x(t)}{OQ}$

or $\quad x(t) = B\sin\omega t = B\sin\dfrac{2\pi}{T}t$

$$= B\sin\frac{2\pi}{30}t$$

8 Given, $y = 3\cos\left(\dfrac{\pi}{4} - 2\omega t\right)$ ...(i)

Velocity, $v = \dfrac{dy}{dt} = 3 \times 2\omega\sin\left(\dfrac{\pi}{4} - 2\omega t\right)$

Acceleration,
$A = \dfrac{dv}{dt} = -4\omega^2 \times 3\cos\left(\dfrac{\pi}{4} - 2\omega t\right)$

As, $A \propto y$ and negative sign shows that it is directed towards equilibrium (or mean position), hence particle will execute SHM. Comparing Eq. (i) with equation $y = r\cos(\phi - \omega' t)$, we have

$$\omega' = 2\omega$$

or $\quad \dfrac{2\pi}{T'} = 2\omega$

or $\quad T' = \dfrac{\pi}{\omega}$

9 $y_1 = a\sin(\omega t)$

$y_2 = b\sin(\omega t + \pi/2)$

$y_R = y_1 + y_2 = \sqrt{a^2 + b^2}\sin(\omega t + \theta)$

SHM with amplitude $\sqrt{a^2 + b^2}$.

10 For a particle executing SHM, acceleration,

$a \propto -\omega^2$ displacement (x) ...(i)

Given, $\quad x = a\sin^2\omega t$ ...(ii)

Differentiating the above equation w.r.t. t, we get

$$\frac{dx}{dt} = 2a\omega(\sin\omega t)(\cos\omega t)$$

Again differentiating, we get

$$\frac{d^2x}{dt^2} = a = 2a\omega^2[\cos^2\omega t - \sin^2\omega t]$$

$$= 2a\omega^2\cos 2\omega t$$

The given equation does not satisfy the condition for SHM [Eq. (i)]. Therefore, motion is not simple harmonic.

11 Here, $y = A\sin\omega t$ and

$$v = A\omega\cos\omega t$$

Since, $\quad \omega = \dfrac{2\pi}{16}$ $\quad [\because T = 16\text{s}]$

Therefore,

$$v = A\left(\frac{2\pi}{16}\right)\cos\frac{2\pi}{16} \times 2$$

$\pi = A \times \dfrac{2\pi}{16} \times \dfrac{1}{\sqrt{2}}$ $\quad [\because v = \pi \text{ m/s}]$

Hence, $A = \pi \times \dfrac{16}{2\pi} \times \sqrt{2} = 8\sqrt{2}$ m

12 Maximum velocity,

$$v_{\max} = a\omega = 3 \times 100 = 300$$

$[\therefore a = 3, \omega = 100]$

13 $v_1^2 = \omega^2(a^2 - x_1^2)$ ...(i)

$v_2^2 = \omega^2(a^2 - x_2^2)$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2),\ \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

14 As we know, the velocity of body executing SHM is given by

$$v = \frac{dx}{dt} = a\omega\cos\omega t = a\omega\sqrt{1 - \sin^2\omega t}$$

$$= \omega\sqrt{a^2 - x^2}$$

Here, $\quad x = \dfrac{a}{2}$

$\therefore \quad v = \omega\sqrt{a^2 - \dfrac{a^2}{4}} = \omega\sqrt{\dfrac{3a^2}{4}}$

$$= \frac{2\pi}{T}\frac{a\sqrt{3}}{2} = \frac{\pi a\sqrt{3}}{T}$$

15 At the mean position, velocity is maximum

$$v_{\max} = a\omega$$

$\omega = \dfrac{v_{\max}}{a} = \dfrac{16}{4} = 4$ $\quad [\because v_{\max} = 16]$

$\therefore \quad v = \omega\sqrt{a^2 - y^2}$

$8\sqrt{3} = 4\sqrt{(4)^2 - (y)^2}$ $\quad [\because v = 8\sqrt{3}]$

$$192 = 16(16 - y^2)$$

$$12 = 16 - y^2$$

$$y = 2 \text{ cm}$$

16 $a = -\omega^2 y \Rightarrow a^2 = \omega^4 y^2$

Hence, $\omega = \sqrt{\dfrac{a}{y}} = \sqrt{\dfrac{2}{0.02}} = \sqrt{100}$

$[\because y = 0.02, a = 2]$

$$= 10 \text{ rad s}^{-1}$$

17 From given graph,
amplitude $(a) = 1$ cm
Time period $(T) = 8$ s

$\therefore \quad \omega = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$ Hz

Acceleration, $A = -\omega^2 a\sin\omega t$

at $\quad t = \dfrac{4}{3}$ s,

$$A = \frac{-\pi^2}{16} \times 1 \times \sin\left(\frac{\pi}{4} \times \frac{4}{3}\right)$$

$\Rightarrow \quad A = \dfrac{-\pi^2}{16}\sin\left(\dfrac{\pi}{3}\right)$

$\Rightarrow \quad A = \dfrac{-\sqrt{3}}{32}\pi^2$ cm/s^2

18 $\quad a = -\omega^2 x$ ...(i)

$\therefore \quad a = -bx$ ...(ii)

$\therefore \quad \omega^2 = b\ ;\ \omega = \sqrt{b}\ ;\ \dfrac{2\pi}{T} = \sqrt{b}$

$\Rightarrow \quad T = \dfrac{2\pi}{\sqrt{b}}$

19 As, $x = A\cos\omega t$

$\therefore \quad v = \dfrac{dx}{dt} = -A\omega\sin\omega t$ ...(i)

and $\ a = \dfrac{d^2x}{dt^2} = -A\omega^2\cos\omega t$ ...(ii)

We can find the correct graph by putting different values of t in Eq. (ii).

At $t = 0, \quad a = -A\omega^2$

At $t = \dfrac{T}{4}$,

$$a = -A\omega^2\cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right) = 0$$

At $t = \dfrac{T}{2}$, $\quad a = -A\omega^2\cos\left(\dfrac{2\pi}{T} \times \dfrac{T}{2}\right)$

$$= -A\omega^2\cos\pi = +A\omega^2$$

At $t = \dfrac{3T}{4}$,

$$a = -A\omega^2\cos\left(\frac{2\pi}{T} \times \frac{3T}{4}\right) = 0$$

At $t = T$,

$$a = -A\omega^2\cos\left(\frac{2\pi}{T} \times T\right) = -A\omega^2$$

This condition is represented by graph in option (c).

20 As we know that, the condition for a body executing SHM is $F = -kx$

So, $\quad a = \dfrac{F}{m} = -\dfrac{k}{m}x$

or $\quad a = -\omega^2 x$

Acceleration $\propto$ – (displacement)

$$A \propto -y$$
$$A = -\omega^2 y$$
$$A = -\frac{k}{m} y$$
$$A = -ky$$

Here, $\quad y = x + a$

$\therefore$ Acceleration $= -k(x + a)$

21 Maximum acceleration of body executing SHM is given by $\alpha_{max} = \omega^2 a$

So, for two different cases,

$$\frac{\alpha_{max_1}}{\alpha_{max_2}} = \frac{\omega_1^2}{\omega_2^2} \quad (\because a \text{ is same})$$

$$= \frac{(100)^2}{(1000)^2} = \frac{1}{10^2}$$

22 The phase difference between the SHM's is $\phi = 0$. Therefore, resultant amplitude is given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$
$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$$
$$= A_1 + A_2 = 6 + 8 = 14$$

23 The acceleration of particle/body executing SHM at any instant (at position x) is given as

$$a = -\omega^2 x$$

where, ω is the angular frequency of the body.

$\Rightarrow \quad |a| = \omega^2 x \quad$ ...(i)

Here, $\quad x = 5\text{m}, |a| = 20\,\text{ms}^{-2}$

Substituting the given values in Eq. (i), we get $\quad 20 = \omega^2 \times 5$

$\Rightarrow \omega^2 = \frac{20}{5} = 4$ or $\omega = 2\,\text{rad s}^{-1}$

As, we know that

Time period, $\quad T = \frac{2\pi}{\omega} \quad$ ...(ii)

$\therefore$ Substituting the value of ω in Eq. (ii), we get

$$T = \frac{2\pi}{2} = \pi\,\text{s}$$

24 Given, when $x = 2$ cm

$$|\mathbf{v}| = |\mathbf{a}|$$

$\Rightarrow \omega\sqrt{A^2 - x^2} = \omega^2 x$

$\Rightarrow \quad \omega = \frac{\sqrt{A^2 - x^2}}{x} = \frac{\sqrt{9-4}}{2}$

$\Rightarrow$ Angular velocity $\omega = \frac{\sqrt{5}}{2}$

$\therefore$ Time period of motion

$$T = \frac{2\pi}{\omega} = \frac{4\pi}{\sqrt{5}}\,\text{s}$$

25 Maximum speed of a particle executing SHM is

$$u_{max} = a\omega = a(2\pi\nu) \Rightarrow \nu = \frac{u_{max}}{2\pi a}$$

Here, $u_{max} = 31.4\ \text{cms}^{-1}$, $a = 5$ cm

Substituting the given values, we have

$$\nu = \frac{31.4}{2 \times 3.14 \times 5} = 1\,\text{Hz}$$

26 For a particle executing SHM, we have maximum acceleration,

$$\alpha = A\omega^2 \quad \text{...(i)}$$

where, A is maximum amplitude and ω is angular velocity of a particle.

Maximum velocity, $\beta = A\omega \quad$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\alpha}{\beta} = \frac{A\omega^2}{A\omega} \Rightarrow \frac{\alpha}{\beta} = \omega = \frac{2\pi}{T}$$

i.e. $T = \frac{2\pi\beta}{\alpha}$

Thus, its time period of vibration,

$T = \frac{2\pi\beta}{\alpha}$.

27 Velocity $v = \omega\sqrt{A^2 - x^2}$

and Acceleration $= \omega^2 x$

Now given, $\omega^2 x = \omega\sqrt{A^2 - x^2}$

$$\omega^2 \cdot 1 = \omega\sqrt{(2)^2 - (1)^2}$$
$$\omega = \sqrt{3}$$

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

28 According to the question, equation of motion of SHM is

$$x = a\sin\left(\omega t + \frac{\pi}{6}\right)$$

velocity of body is given by

$$v = \frac{dx}{dt} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\frac{a\omega}{2} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\frac{1}{2} = \cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\omega t + \frac{\pi}{6} = \frac{\pi}{3} \quad \left[\because \cos\frac{\pi}{3} = \frac{1}{2}\right]$$

$\Rightarrow \quad \omega t = \frac{\pi}{6}$

$$\frac{2\pi}{T}t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$$

29 Let $\quad y = A\sin\omega t$

i.e. $\quad \frac{A}{\sqrt{2}} = A\sin\omega t$

$\Rightarrow \quad \sin\omega t = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

i.e. $\quad \omega t = \frac{\pi}{4}$

But $\quad \omega = \frac{2\pi}{T}$

Therefore, $\quad t = \frac{T}{8}$

30 Here, $y = A\cos\omega t$.

When $y = A/2$, we find

$\cos\omega t = \frac{1}{2} = \cos\frac{\pi}{3}$.

Hence, $\omega t = \frac{\pi}{3}$

or $\quad t = \frac{\pi}{3\omega} = \frac{\pi}{3 \times 2\pi/T} = \frac{T}{6}$

31
$$K = 1/2\, m\omega^2\,[A^2 - y^2]$$
$$= \frac{1}{2}m\omega^2 A^2\left[1 - \frac{y^2}{A^2}\right]$$
$$= \left[1 - \frac{y^2}{A^2}\right]E = \frac{3}{4}E$$

32 Here, $\quad$ KE = PE

$$\frac{1}{2}m\omega^2(A^2 - y^2) = \frac{1}{2}m\omega^2 y^2$$

$$y = \frac{A}{\sqrt{2}} \Rightarrow y = 2\sqrt{2}\,\text{cm}$$

33 We know that, $T = 2\pi\sqrt{\frac{l}{g}}$

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T' = 2\pi\sqrt{\frac{4l}{g}}$$

$\Rightarrow \quad \frac{T'}{T} = 2 \Rightarrow T' = 2T$

When l is made four times, the time period is doubled.

34 The acceleration due to gravity at a depth d is

$g_d = g\left[1 - \frac{d}{R}\right] = g\left[\frac{R-d}{R}\right]$. But

$R - d = y$ is the distance of the point from the centre of the earth. Then,

$g_d = \frac{g}{R}y$. So, the acceleration of the body in the tunnel is proportional to y. And the motion is SHM with time period

$$T = 2\pi\left[\frac{R}{g}\right]^{1/2}$$

Hence, it oscillates between the two ends of the tunnel.

35 Frequency, $n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ or $n \propto \frac{1}{\sqrt{L}}$

$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{L_2}{2L_2}}, \quad \frac{n_1}{n_2} = \frac{1}{\sqrt{2}}$

$$n_2 = \sqrt{2}n_1$$

$\Rightarrow \quad n_2 > n_1$

Energy, $E = \frac{1}{2}m\omega^2 a^2 = 2\pi^2 m n^2 a^2$

and $\quad a^2 \propto \frac{1}{mn^2}$ $\quad$ [$\because E$ is same]

$$\therefore \quad \frac{a_1^2}{a_2^2} = \frac{m_2 n_2^2}{m_1 n_1^2}$$

Given, $n_2 > n_1$ and $m_1 = m_2 \Rightarrow a_1 > a_2$
So, amplitude of B smaller than A.

36 Potential energy of a simple harmonic oscillator

$$U = \frac{1}{2} m\omega^2 x^2$$

Kinetic energy of a simple harmonic oscillator

$$K = \frac{1}{2} m\omega^2 (a^2 - x^2)$$

Here, x = Displacement from mean position

a = Maximum displacement (or amplitude) from mean position

Total energy is $E = U + K$

$$= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 (a^2 - x^2) = \frac{1}{2} m\omega^2 a^2$$

When the particle is half way to its end point i.e. at half of its amplitude, then

$$x = \frac{a}{2}$$

Hence, potential energy

$$U = \frac{1}{2} m\omega^2 \left(\frac{a}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2} m\omega^2 a^2\right)$$

$$\Rightarrow U = \frac{E}{4}$$

37 As we know that

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

Case I $\quad T_1 = 2\pi\sqrt{\frac{m}{k}}$ $\quad$ ...(i)

Case II When the mass m is increased by 1 kg, then, $T_2 = 2\pi\sqrt{\frac{m+1}{k}}$ $\quad$ ...(ii)

From Eqs. (ii) and (i), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \frac{5}{3} = \sqrt{\frac{m+1}{m}} \Rightarrow \frac{25}{9} = \frac{m+1}{m}$$

$$\Rightarrow \frac{25}{9} = 1 + \frac{1}{m} \quad \Rightarrow \frac{1}{m} = \frac{16}{9}$$

$$\therefore \quad m = \frac{9}{16} \text{ kg}$$

38 By the law of conservation of energy, Kinetic energy of mass = Energy stored in spring

i.e. $\frac{1}{2} mv^2 = \frac{1}{2} kx^2$

$$\therefore \quad x^2 = \frac{mv^2}{k} \Rightarrow x = \sqrt{\left(\frac{mv^2}{k}\right)}$$

$$\Rightarrow \quad x = \sqrt{\left(\frac{0.5 \times 1.5 \times 1.5}{50}\right)} = 0.15 \text{ m}$$

39 Effective force constant in case (iii) and (iv) is $K_{\text{eff}} = 2k + 2k = 4k$

Therefore, $\quad T_1 = T_2 = 2\pi\sqrt{\frac{m}{k}}$

and $T_3 = T_4 = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$

40 Time period of the system is

$$T = 2\pi\sqrt{\frac{m_1 + m_2}{k}}$$

Given, $m_1 = 6$ kg,
$m_2 = m$ = mass of girl
$k = 600$ N/m, $T = 0.758$ s

$$\therefore T = 2\pi\sqrt{\frac{6+m}{600}} = 0.758$$

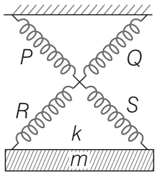
On squaring both sides, we get

$$\frac{6+m}{600} = \frac{(0.758)^2}{4\pi^2} \Rightarrow m = 2.74 \text{ kg}$$

41 For series combination of springs,

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}, \quad k = \frac{k_1 k_2}{k_1 + k_2}$$

42



Spring P and Q, R and S are in parallel.

Then, $\quad x = k + k = 2k$ $\quad$ [for P, Q]

and $\quad y = k + k = 2k$ $\quad$ [for R, S]

x and y both in series

$$\therefore \quad \frac{1}{k'} = \frac{1}{x} + \frac{1}{y} = \frac{1}{k}$$

Time period, $T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{k}}$

43 The fundamental frequency of string

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{k}{l}$$

$$\therefore \quad \nu_1 l_1 = \nu_2 l_2 = \nu_3 l_3 = k \quad \text{...(i)}$$

From Eq. (i) $l_1 = \frac{k}{\nu_1}, l_2 = \frac{k}{\nu_2}, l_3 = \frac{k}{\nu_3}$,

Original length $\; l = \frac{k}{\nu}$

Here, $\quad l = l_1 + l_2 + l_3$

$$\frac{k}{\nu} = \frac{k}{\nu_1} + \frac{k}{\nu_2} + \frac{k}{\nu_3}$$

$$\frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}.$$

44 By using $k \propto \frac{1}{l}$

Since, one-fourth length is cut away, so remaining length is $\frac{3}{4}$th, hence, k becomes $\frac{4}{3}$ times i.e. $k' = \frac{4}{3} k$.

45 Time period of spring pendulum

$$T = 2\pi\sqrt{\frac{M}{k}}$$

When mass is doubled,

$$T' = 2\pi\sqrt{\frac{2M}{k}}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{2}$$

$$\Rightarrow \quad T' = \sqrt{2}T$$

Hence, option (d) is true.

46 $T = 2\pi\sqrt{\frac{m}{k}}$ $\cdot$ Since, $T = \frac{1}{n}$

Therefore, $\quad n \propto \sqrt{k}$

Also, for a spring $k \propto \frac{1}{x}$

Thus, $\quad \frac{k_2}{k_1} = \frac{x_1}{x_2} = 2$

Therefore, new frequency

$$n_2 = n_1 \times \sqrt{\frac{k_2}{k_1}} = n\sqrt{2}$$

47 Amplitude of damped oscillator

$$A = A_0 e^{-\lambda t}; \lambda = \text{constant}, t = \text{time}$$

For $t = 1$ min,

$$\frac{A_0}{2} = A_0 e^{-\lambda t} \Rightarrow e^{\lambda} = 2$$

For $t = 3$ min,

$$A = A_0 e^{-\lambda \times 3} = \frac{A_0}{(e^{\lambda})^3} = \frac{A_0}{(2)^3} = \frac{A_0}{X}$$

$$\Rightarrow X = (2)^3$$

48 Given, Damping force $\propto$ velocity

$$F \propto v$$

$$\Rightarrow \quad F = kv \quad \Rightarrow k = \frac{F}{v}$$

Unit of $k = \frac{\text{unit of } F}{\text{unit of } v} = \frac{\text{kg - ms}^{-2}}{\text{ms}^{-1}}$

$= \text{kg s}^{-1}$

SESSION 2

1 Let the minimum amplitude of SHM is a. Restoring force on spring

$$F = ka$$

$$\therefore \quad ka = mg \text{ or } a = \frac{mg}{k}$$

Here, $m = 2$ kg, $k = 200 \text{ Nm}^{-1}$,
$g = 10 \text{ ms}^{-2}$

$$\therefore \quad a = \frac{2 \times 10}{200} = \frac{10}{100} \text{ m}$$

$$= \frac{10}{100} \times 100 \text{ cm} = 10 \text{ cm}$$

Hence, minimum amplitude of the motion should be 10 cm, so that the mass gets detached from the pan.

2 $\because \quad x = a\sin\omega t$

$\therefore \sin\omega t = \dfrac{x}{a}$

$\cos\omega t = \dfrac{\sqrt{a^2 - x^2}}{a}$

and $y = a\sin 2\omega t = 2a\sin\omega t\cos\omega t$

$= 2a\,\dfrac{x}{a}\,\dfrac{\sqrt{a^2 - x^2}}{a} = \dfrac{2x\sqrt{a^2 - x^2}}{a}$

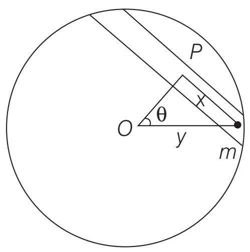
$\therefore y^2 = \dfrac{4x^2}{a^2}(a^2 - x^2)$

3 Force on the mass m along the tunnel will be

$F = -\left(\dfrac{GMm}{R^3}\,y\right)\sin\theta$

$\left[\text{From fig. } \sin\theta = \dfrac{x}{y}\right]$

$= -\left(\dfrac{GMm}{R^3}\,y\right)\left(\dfrac{x}{y}\right) = -\dfrac{GMm}{R^3}\,x$



or $\quad F = -\dfrac{mg}{R}\,x \qquad \left[\because g = \dfrac{GM}{R^2}\right]$

It is a SHM with $k = mg/R$

Hence, $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{R}{g}}$,

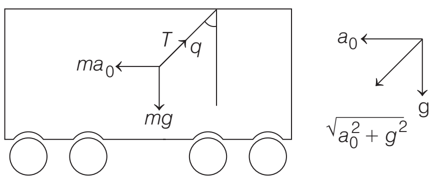
where g is the acceleration on the surface of the earth.

4 Frequency, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{BA^2}{MV_0}}$

B = bulk modulus of elasticity

$T = 2\pi\sqrt{\dfrac{MV_0}{BA^2}} = 2\pi\sqrt{\dfrac{M(hA)}{pA^2}} = 2\pi\sqrt{\dfrac{Mh}{pA}}$

5 Let train is moving towards right. The Pseudo force is acting in left direction $= ma_0$



Effective acceleration,

$a_{\text{eff}} = \sqrt{a_0^2 + g^2}$

$T = 2\pi\sqrt{\dfrac{l}{a_{\text{eff}}}} = 2\pi\sqrt{\dfrac{l}{\sqrt{a_0^2 + g^2}}}$

6 Force constant of a wire is

$K = \dfrac{YA}{l}$ or $T = 2\pi\sqrt{\dfrac{m}{K}} = 2\pi\sqrt{\dfrac{ml}{YA}}$

7 $K_{\text{eff}} = 2k$

$\therefore T = 2\pi\sqrt{\dfrac{M}{2k}}$

Period (or K_{eff}) is independent of θ.

8 Let block is displaced through x, then weight of displaced water or upthrust (upwards) $= -A\,x\rho g$

where, A is area of cross-section of the block and ρ is its density. This must be equal to force ($= ma$) applied, where m is mass of the block and a is acceleration.

$\therefore \quad ma = -A\,x\rho g$

or $\quad a = -\dfrac{A\rho g}{m}\,x = -\omega^2 x$

This is the equation of SHM.

Time period of oscillation

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{A\rho g}} \Rightarrow T \propto \dfrac{1}{\sqrt{A}}$

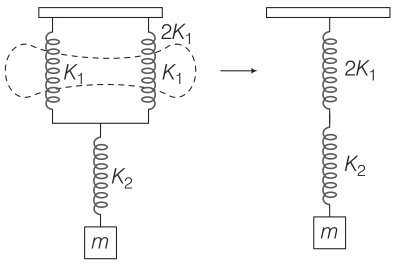
9 Extensions in springs are x_1 and x_2, then

$k_1x_1 = k_2x_2$ and $x_1 + x_2 = A$

$\Rightarrow \quad x_2 = \dfrac{k_1x_1}{k_2} \Rightarrow x_1 + \dfrac{k_1x_1}{k_2} = A$

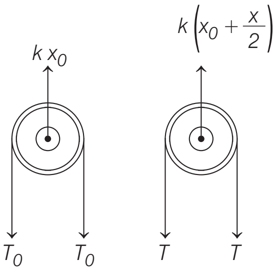
$\Rightarrow \quad x_1 = \dfrac{k_2A}{k_1 + k_2}$

10 In series combination



$\dfrac{1}{k_S} = \dfrac{1}{2k_1} + \dfrac{1}{k_2} \Rightarrow k_S = \left[\dfrac{1}{2k_1} + \dfrac{1}{k_2}\right]^{-1}$

11 Let tension in spring at equilibrium positions for block is T_0



$\therefore \quad mg = T_0$

From force diagram of pulley,

$2T_0 = kx_0 \Rightarrow 2mg = kx_0$

When block is displaced a small distance x in downward direction from equilibrium position, tension in the string is T.

$mg - T = ma$

$2T = k\left(x_0 + \dfrac{x}{2}\right) = kx_0 + \dfrac{kx}{2}$

$mg - \left(mg + \dfrac{kx}{4}\right) = ma$

$-\dfrac{kx}{4} = ma$

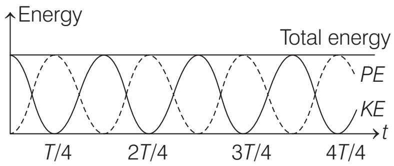
$\Rightarrow \quad a = -\left(\dfrac{k}{4m}\right)x$

$\Rightarrow \quad \omega^2(-x) = -\left(\dfrac{k}{4m}\right)x$

$\omega = \sqrt{\dfrac{k}{4m}}$

$\Rightarrow \quad \dfrac{2\pi}{T} = \sqrt{\dfrac{k}{4m}} \Rightarrow T = 4\pi\sqrt{\dfrac{m}{k}}.$

12 In SHM, the total energy of the particle is constant at all instants which is totally kinetic when particle is passing through the mean position and is totally potential when particle is passing through the extreme position.



The variation of PE and KE with time is shown in figure by dotted parabolic curve and solid parabolic curve, respectively.

Figure indicates that maximum values of total energy, KE and PE of SHM are equal.

Now, $\quad E_K = K_0\cos^2\omega t$

$\therefore \quad (E_K)_{\max} = K_0$

So, $\quad (E_P)_{\max} = K_0$

and $\quad (E)_{\text{total}} = K_0$

13 From conservation principle,

$mgh = \dfrac{1}{2}k\,x_0^2 - mgx_0$

where, x_0 is maximum elongation in spring.

$\Rightarrow \quad \dfrac{1}{2}kx_0^2 - mgx_0 - mgh = 0$

$\Rightarrow \quad x_0^2 - \dfrac{2mg}{k}x_0 - \dfrac{2mg}{k}h = 0$

$x_0 = \dfrac{\dfrac{2mg}{k} \pm \sqrt{\left(\dfrac{2mg}{k}\right)^2 + 4\times\dfrac{2mg}{k}h}}{2}$

Amplitude = elongation in spring for lowest extreme position – elongation in spring for equilibrium position

$$= x_0 - x_1 = \frac{mg}{k}\sqrt{1 + \frac{2hk}{mg}}$$

$$\left[\because x_1 = \frac{mg}{k}\right]$$

Hence, option (b) is true.

14 Let $y = A \sin \omega t$

Then, $K = \frac{1}{2} MA^2\omega^2 \cos^2 \omega t$ and

$$U = \frac{1}{2} MA^2 \omega^2 \sin^2 \omega t$$

Therefore,

$$\frac{K}{U} = \frac{\cos^2 \omega t}{\sin^2 \omega t} = \frac{1 - \sin^2 \omega t}{\sin^2 \omega t}$$

$$= \frac{1 - y^2/A^2}{y^2/A^2}$$

$$= \frac{A^2 - y^2}{y^2}$$

15 Potential energy of stretched string is

$$U = \frac{1}{2} kx^2$$

where, k is spring constant or force constant.

$$\therefore \quad \frac{U_1}{U_2} = \frac{x_1^2}{x_2^2} \quad \text{...(i)}$$

Given, $U_1 = U$, $x_1 = 2$ cm, and $x_2 = 8$ cm

On putting these values in Eq. (i), we get

$$\frac{U}{U_2} = \frac{(2)^2}{(8)^2} = \frac{4}{64} = \frac{1}{16}$$

$$\therefore \quad U_2 = 16\,U$$

16 Mass of the particle = m

Spring constant = k

The time period of oscillator, $T = 2\pi\sqrt{\frac{m}{k}}$

As $k \propto \frac{1}{l}$ [where, l is the length of spring]

$$k' = 2k$$

$$\therefore \quad T' = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}} T$$

17 $x = A \cos \omega t$

Displacement in t time = $A - A\cos \omega t$

For $t = \tau \Rightarrow A[1 - \cos \omega t] = a$ ...(i)

For $t = 2\tau \Rightarrow A[1 - \cos 2\omega t] = 3a$ ...(ii)

Divide eq (i) by eq (ii)

$$\frac{1 - \cos \omega t}{1 - \cos 2\omega t} = \frac{1}{3}$$

$$\frac{1 - \cos \omega t}{2\sin^2 \omega t} = \frac{1}{3}$$

say $x = \cos \omega t$

$$\frac{1 - x}{2(1 - x^2)} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{2(1 + x)} = \frac{1}{3}$$

$$\Rightarrow \quad 3 = 2 + 2x$$

$$\Rightarrow \quad x = \frac{1}{2} = \cos \omega t$$

$$A = 2a, \; \omega t = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$$\Rightarrow \quad T = 6\tau$$